

Buchmüller Scaling, the QCD Pomeron, and Tevatron Diffractive Hard Scattering

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Abstract

We discuss the observed scaling of the small- x diffractive and total deep-inelastic structure functions at HERA. We argue that the parton interpretation of Buchmüller and Hebecker can be understood within QCD as the appearance of the Pomeron in a Super-Critical phase. In diffractive hard scattering, the Pomeron appears as reggeized gluon exchange in a color-compensating background field. The formalism can also be applied to diffractive W production at the Tevatron. If the scaling is a true asymptotic property then it should anticipate the appearance of a further massive sector of QCD and associated asymptotic Critical Pomeron behaviour.

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1. INTRODUCTION

Presently diffractive physics is thought to be non-perturbative and uncalculable within QCD. It is hoped, however, that in limited kinematic circumstances involving small- x and large Q^2 , semi-perturbative BFKL Pomeron calculations[1] will be applicable. Unfortunately, the BFKL Pomeron is a complicated multi-gluon phenomenon which, from the experimental point of view, has proven difficult to isolate. In contrast, Buchmüller[2] has noticed a remarkably simple scaling relation between the diffractive and full deep-inelastic cross-sections measured at small- x at HERA. Essentially, both cross-sections have the same Q^2 and x -dependence[3]. This is illustrated in Fig. 1.1 using the ZEUS data but the H1 data give the same result.

ZEUS 1993

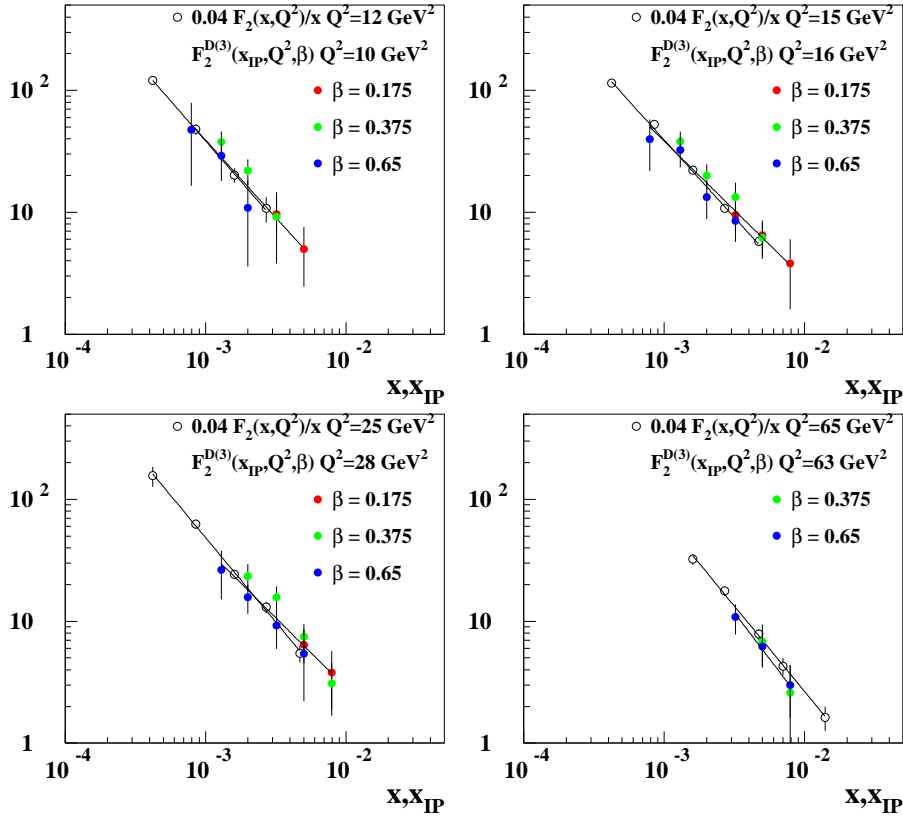


Fig. 1.1 Scaling of the Diffractive and Full Deep-Inelastic Structure Functions - the notation is that of Section 2.

The scaling of Fig. 1.1 suggests an extraordinary simplicity for the Pomeron at large Q^2 - in a sense (that we describe below) *it acts like a single gluon*. Our purpose in this paper is both to explain this simplicity in the context of our previous study of the QCD Pomeron[4, 5], and to expand the picture to hadronic diffractive hard scattering. (It is many years since we first pointed out[6] that, in a “Super-Critical Phase”, the QCD Pomeron would appear like a single *reggeized* gluon.) Conventionally, the deep-inelastic diffractive cross-section is believed to contain both non-perturbative “soft Pomeron” physics and BFKL contributions, while the full cross-section is usually evaluated perturbatively (with BFKL evolution included). Even if the diffractive cross-section were completely given[7] by BFKL, no simple scaling relation between the two cross-sections would be anticipated.

There may be various models that give the proposed scaling behavior either directly or approximately[8]. Also it may appear less significant experimentally as the accuracy of the data improves. However, in a recent paper[9] Buchmüller and Hebecker (BH) have made an attractive proposal that we would like to take very seriously. They suggest that the scaling is a direct consequence of the rapidity gap (diffractive) events originating from the same small- x parton scattering process as the normal deep-inelastic events, i.e. the photon-gluon fusion process of Fig. 1.2.

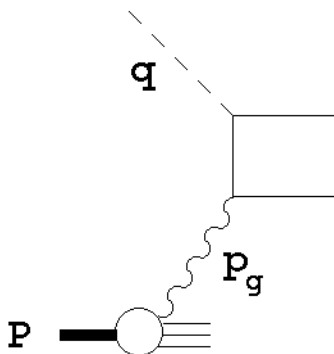


Fig. 1.2 Photon-Gluon Fusion

More importantly, BH also suppose that the only subsequent interaction is very simple. It is pictured as a rescattering in a “classical” color field, formed by wee-partons in the proton, which randomly rotates the color of the quark-antiquark pair. The probability that the quark final state is color neutral (and so produces the diffractively excited final state) is then 1/9 th. This successfully predicts the experimental

ratio of the diffractive cross-section to the total cross-section and the scaling follows almost trivially. The Pomeron that is exchanged is clearly a simple gluon - plus only a “color correction”.

In this paper we will discuss how the, at first sight paradoxical, simplicity of the BH picture can be understood directly within QCD if the Pomeron is in a Super-Critical phase of Reggeon Field Theory. (Although we will replace $1/9$ th by $1/8$ th!) Indeed the basis for our analysis[4] of the Pomeron in QCD is to initially identify this phase in a gauge theory framework. Part of the purpose of this paper will be to discuss how the first approximation, in which the Pomeron is a single (reggeized) gluon, is exposed by the deep-inelastic limit.

There must, of course, be a compensation for the exchanged color of the gluon. In our work there is a “reggeon condensate” which plays a role very similar to the classical wee-parton field envisaged in the BH picture. The reggeon condensate arises from infra-red divergences of Regge limit transverse momentum diagrams involving massless quarks. However, we believe it should also have an interpretation as a classical field related to the regularization of (multi-)quark operators defining physical states at infinite momentum. The massless fermion axial U(1) anomaly is crucial for this and we suspect that the Super-Critical Pomeron can be associated with a generalized “winding-number condensate” arising, essentially, from instanton interactions. In analogy with the role of the anomaly in the Schwinger model[10], this condensate can be viewed as producing confinement and chiral symmetry breaking *at high-energy*. It is not an SU(3) invariant but rather is an octet (resulting from the restriction of SU(3) instantons to an SU(2) subgroup). Therefore, in the Super-Critical phase, an infinite momentum hadron is an octet quark state in the presence of the condensate. Gauge-invariance requires an averaging over all SU(3) directions. Nevertheless, within this averaging, it is possible for single gluon exchange between the quarks to be accompanied by an (instanton) interaction which simply realigns the direction of the condensate. This is the underlying idea in our version of the BH picture. For the Pomeron, the net color exchanged, by the combination of a gluon and an instanton must be zero. In a general perturbative gluon interaction this need not be the case and the result is a factor of $1/8$ th.

We will see that as the Pomeron, in a sense, becomes simpler at the partonic level, it also becomes a more “non-perturbative” phenomenon in which confinement, and not just the requirement of color zero exchange, plays an essential role. The Super-Critical condensate is a confinement mechanism which is operative at the scale of the hard scattering. It is related to, but goes much further than, the color zero

cancelation of infra-red divergences. We find that if the Pomeron is to couple to a hard scattering process, the parton final state created must be converted to a true hadronic state by the condensate. This is straightforwardly the case for the quark-antiquark state of Fig. 1.2, but in general is a strong constraint on the possible parton states involved. A particular consequence is that the Ingelman-Schlein assumption[11] of a universal hadronic parton distribution function for the Pomeron is not valid. Indeed, when our formalism is extended to hadronic diffractive W and jet production, the confinement requirement actually resolves what appear to be puzzling features of the Tevatron data when the Ingelman-Schlein formalism is used[12]. Since it is a simple Regge pole, there is also an obvious simplification of the theoretical factorization properties of the hard Pomeron within the Super-Critical framework. In general it seems that a simpler and more unified picture of diffractive physics and the Pomeron may emerge if we develop the BH picture by exploiting our formalism.

The Super-Critical Pomeron is a (transverse momentum) cut-off dependent construction and as the effective cut-off increases with energy, it will eventually disappear unless the true asymptotic behavior of the theory is given by the Critical Pomeron[13]. Therefore deep-inelastic diffractive scaling, or “Buchmüller scaling”, can occur as an *exact asymptotic property* only in this very special circumstance. In fact we have argued for a long time that the Critical Pomeron may be the only way to obtain self-consistent unitary asymptotic behavior in the Regge limit and that a necessary higher-mass quark sector[4] should therefore exist. It may seem surprising that an asymptotic Pomeron property could be directly related to deep-inelastic scaling at the relatively low energy scale of HERA. However, if we take the BH picture seriously and ask under what circumstances it could provide the underlying physical explanation of an asymptotic scaling property, this is what we are led to.

Unfortunately many of the concepts involved in our work are either unfamiliar or complicated and technically it is certainly incomplete. As a result we will give only an outline in this paper that concentrates on illustration of the ideas involved. Our hope is that a connection with the BH work will help demonstrate the underlying simplicity and physical relevance of the picture of the Pomeron that we have arrived at, and also lead to further applications.

We will break our discussion down into relatively small Sections, beginning with a summary of the BH paper in Section 2. In Section 3 we will, very briefly, review what conventional QCD/Pomeron physics has to say about deep-inelastic diffractive scaling. Three Sections follow that are self-contained descriptions of crucial elements in our argument. In Section 4 we discuss the conditions under which asymptotic

freedom can be combined with gluon mass generation via the Higgs mechanism. In Section 5 we describe the analysis of Regge limit infra-red divergences and the resulting Pomeron when the gauge symmetry of QCD is broken to SU(2), together with the conditions for the Critical Pomeron to occur. Section 6 contains a very short description of the Super-Critical Pomeron together with a discussion of the role of the cut-off in the phase-transition. This allows us to discuss how the Super-Critical phase occurs in conventional QCD. We return to deep-inelastic scattering in Section 7 and discuss how the BH picture is realised in QCD. Section 8 contains our discussion of diffractive hard scattering in hadron-hadron scattering. Section 9 is a final comment on the possible new physics at higher-energy scales that is implied if “Buchmüller scaling” is a true asymptotic property.

2. THE BUCHMÜLLER AND HEBECKER FORMALISM

We begin with a very brief summary of the BH paper. Starting from the quark-antiquark pair production of Fig. 1.2, the contribution to the inclusive structure function $F_2(x, Q^2)$ is (in the “massive gluon” scheme[14])

$$\Delta^{(g)} F_2(x, Q^2) = x \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} g(\xi) F(\beta, Q^2) . \quad (2.1)$$

where $g(x)$ is the gluon density and, if we define momenta as in Fig. 1.2 so that

$$Q^2 = -q^2 , \quad x = \frac{Q^2}{2P \cdot q} , \quad M^2 = (q + p_g)^2 . \quad (2.2)$$

and write $\vec{p}_g = x_{\mathbb{P}} \vec{P}$ so that when $-p_g^2 = m_g^2 \ll Q^2$, M^2

$$\beta \equiv \frac{Q^2}{Q^2 + M^2} \simeq \frac{x}{x_{\mathbb{P}}} . \quad (2.3)$$

we have

$$F(\beta, Q^2) = \left((\beta^2 + (1 - \beta)^2) \ln \frac{Q^2}{m_g^2 \beta^2} - 2 + 6\beta(1 - \beta) \right) . \quad (2.4)$$

Assuming that $g(x)$ is power-behaved at small x leads to a simple approximation for F_2 , i.e.

$$F_2(x, Q^2) \simeq \frac{\alpha_s}{3\pi} \sum_q e_q^2 x g(x) \left(\frac{2}{3} + \ln \frac{Q^2}{m_g^2} \right) . \quad (2.5)$$

which (with a simple parameterization for $g(x)$ and with $m_g \sim 1$ GeV) fits the data very well .

In fact (2.5) works much better than it should since, in principle, it only describes that part of the cross-section in which the final state contains two (quark) jets. That kinematically either the quark or the antiquark is likely to be relatively soft corresponds, of course, to the experimental feature that the bulk of the cross-section contains only one jet. If the soft quark (anti-quark) is produced by the photon, the corresponding contribution to the photon structure function would simply be given by the appropriate part of Fig. 1.2. A similar remark would apply when the soft quark (anti-quark) is produced by the gluon if this carries large enough momentum transfer. As the momentum transfer becomes smaller, we might expect the single gluon to be replaced by a (multi-gluon) Pomeron. If the gluon in Fig. 1.2 merges smoothly into the Pomeron, as we are about to describe, this may actually be connected with the ability of (2.5) to approximate far more of the cross-section than it naively should. This point will be important later.

The assumption that diffractive deep-inelastic scattering is described by the same process as pictured in Fig. 1.2, except that the quark-antiquark pair evolves into a color singlet state with probability $1/9$, predicts[9] that

$$F_2^D(x_{\mathbb{P}}, Q^2, \beta) \simeq \frac{1}{9} \frac{\alpha_s}{2\pi} \sum_q e_q^2 g(x_{\mathbb{P}}) \bar{F}_2^D(\beta, Q^2) , \quad (2.6)$$

where

$$\bar{F}_2^D(\beta, Q^2) = \beta F(\beta, Q^2) \quad (2.7)$$

Since both $g(x_{\mathbb{P}})$ and m_g have been determined by the fit to the inclusive structure function F_2 , the diffractive structure function is unambiguously predicted, including its normalization. For β between 0.2 and 0.6, $\bar{F}_2^D(\beta, Q^2)$ varies rather slowly with β . As a result, in this interval,

$$F_2^D(x_{\mathbb{P}}, Q^2, \beta) \simeq \frac{C}{x_{\mathbb{P}}} F_2(x_{\mathbb{P}}, Q^2) , \quad (2.8)$$

where $C \simeq 0.04$, independent of Q^2 . This is the scaling relation, first suggested by Buchmüller[2], which is tested in Fig. 1.1. ($F_2^{D(3)}$ should be identified with F_2^D). We conclude that the BH model provides a very successful description of the properties of the large rapidity gap deep-inelastic events when x is small and Q^2 and M^2 are large - so that the cross-section is relatively *insensitive to the gluon mass m_g* .

Note, however, that the gluon mass has played two essential roles. Firstly it has provided the scale for the Q^2 scaling violation. Secondly, without a gluon mass, higher-order corrections which reggeize the gluon exchange producing the diffractive cross-section would be infra-red divergent - providing a strong exponential suppression of this exchange.

3. CONVENTIONAL QCD/POMERON PHYSICS

To put the following discussion in context, we briefly outline what conventional QCD/Pomeron physics has to say about the diffractive cross-section. Suppose first that large Q^2 produces large transverse momentum exchange across the rapidity gap, so that perturbation theory may be applicable. The simplest color-zero “perturbative Pomeron”, which avoids the exponential suppression by infra-red divergences, is two gluons. To calculate scaling violations, with a conventional factorization scale replacing the gluon mass, a factorization theorem must be proved. Attempts to derive such a theorem have led to various problems[15, 16] and indeed the factorization properties are different in deep-inelastic and hadron scattering. (This actually reflects the fact that the “Pomeron” involved is not a Regge pole.) To the extent that there is factorization, a new parton distribution appears[16], not the gluon distribution of the BH picture.

If the issue of the Q^2 scaling violations is ignored, and one looks instead at the small- x dependence, higher-order corrections lead to the reggeization of both gluons, together with reggeon interactions, and the BFKL Pomeron is formed across the rapidity gap. This is illustrated in Fig. 3.1 and discussed at length in [7].

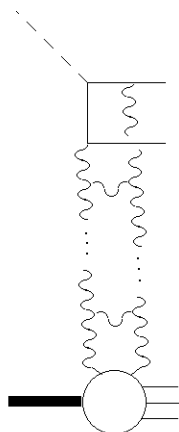


Fig. 3.1 BFKL Pomeron Production of a Rapidity Gap

If we compare with the calculation of the total deep-inelastic cross-section via BFKL then higher-order corrections, producing the square of the gluon distribution are involved. This implies that the diffractive structure function should rise faster at small- x than the full structure function.

It is anticipated, of course, that the bulk of the diffractive cross-section involves relatively small momentum transfer across the rapidity gap, even when Q^2 and M^2 are large. We then expect the “soft Pomeron” to be involved. The conventional (Ingelman and Schlein[11]) assumption is that the large Q^2 is absorbed by a parton interaction described by a Pomeron structure function. Since non-perturbative physics is invoked there is no reason to expect any very simple relation with respect to the perturbatively calculated total cross-section.

Note that, at any fixed Q^2 , the asymptotic small- x behaviour of the total cross-section is, of course, given by the full non-perturbative Pomeron. In this case the scaling behavior we are looking for requires that the x -dependence (i.e the cross-section increase with energy) be the same for the one Pomeron exchange that gives the total cross-section and the triple-Pomeron interaction that gives the high-mass rapidity-gap cross-section. This is a consistency condition for the asymptotic Pomeron that is satisfied[17], so far as is known, only by the Critical Pomeron. This makes it plausible that the appearance of combined deep-inelastic and diffractive scaling may actually be the short-distance precursor of Critical Pomeron asymptotic behavior.

4. A GLUON MASS FROM SYMMETRY BREAKING

A simple prerequisite for the BH calculation would be that a gluon mass can be smoothly introduced into the calculation of scaling violations without destroying asymptotic freedom. To introduce a gluon mass in a gauge-invariant way requires the use of the Higgs mechanism. A-priori, we then expect to lose contact with confining, asymptotically-free, unbroken QCD. However, as observed in the original asymptotic freedom paper of Gross and Wilzcek[18], we can break the symmetry from $SU(3)$ to $SU(2)$ by using a single fundamental representation Higgs scalar. In this special case, asymptotic freedom of both the gauge coupling and the scalar self-coupling can be preserved *if enough fermions are present*, as we briefly illustrate.

If $g(t)$ and $h(t)$ are respectively the gauge and scalar scale-dependent couplings, then

$$\frac{dg}{dt} = -\frac{1}{2}b_0t^3 + \dots \quad (4.1)$$

where

$$b_0 = \frac{1}{8\pi^2} \left[11 - \frac{4}{3} S_3(f) - \frac{1}{6} \right] \quad (4.2)$$

The $1/6$ is due to the triplet scalar and $S_3(f)$ depends on the number (and representation) of the quarks present. Similarly

$$\frac{dh}{dt} = Ah^2 + Bg^2 + Cg^4 + \dots \quad (4.3)$$

where

$$A = \frac{7}{8\pi^2}, \quad B = -\frac{1}{\pi^2} \quad \text{and} \quad C = \frac{13}{48\pi^2}. \quad (4.4)$$

Requiring that $h \sim g^2 \rightarrow 0$ consistently gives a stability equation which requires that b_0 be very small i.e.

$$\frac{5}{24} > 8\pi^2 b_0 \quad (4.5)$$

for a solution.

(4.5) is satisfied if, and only if, the maximum number of fermions consistent with asymptotic freedom are present, i.e. $N_f = N_f^{max}$. In order to relax this condition we will need to introduce Reggeon Field Theory explicitly into our discussion, as we will do shortly. This will enable us to understand the general significance, for the Pomeron, of breaking the gauge symmetry from $SU(3)$ to $SU(2)$, independently of the number of fermions present. For the moment we note that the large number of fermions is a prerequisite only if we insist on combining asymptotic freedom with the symmetry breaking generating the gluon mass. Of course, the additional fermions can be present at a higher mass scale. The effective theory obtained by integrating them out will be asymptotically free and the gluon mass can be smoothly generated as we have discussed. Note also that as the gluon mass is taken to zero, the scalar mass can be simultaneously sent to infinity so that it decouples from the theory. Because it is in the fundamental representation the complementarity principle applies to this decoupling, i.e. there is a smooth relation between the ‘‘Higgs region’’ of parameter space and the ‘‘confinement region’’. A related property is that a gauge-invariant description of the ‘‘symmetry breaking’’ can be given. A full discussion can be found in [4].

The spectrum of gluons when the symmetry is broken is important for our discussion. Under the remaining $SU(2)$ symmetry we have

- one massless triplet

- two massive doublets
- one massive singlet

giving, of course, eight in total. The massless triplet is responsible for the remaining SU(2) gauge symmetry while the singlet is the essential feature for our purposes.

5. INFRA-RED DIVERGENCES AND THE WEE-PARTON CONDENSATE

An (extremely) non-trivial part of our discussion is the emergence of a reggeon, or “wee-parton”, condensate when the gauge symmetry is broken as discussed in the last Section. It underlies our understanding of the Super-Critical Pomeron and is the basis for our argument that a semi-classical field can indeed be responsible for color compensation of parton scattering, as in the BH picture. The theoretical problem of determining conditions under which the wee partons in a hadron can be described by such a field may appear to be novel. However, it first arose nearly twenty years ago as the problem of making sense of the notion of a “Pomeron condensate” defining the Super-Critical Reggeon Field Theory[5] that we discuss in the next Section.

According to our analysis[4], if the gauge symmetry is broken from SU(3) to SU(2) then, *in the diffractive scattering regime*, all the infra-red divergences of reggeon diagrams due to the remaining massless gluons can be absorbed in a stable “reggeon condensate”. If this symmetry-breaking is to be a smooth effect at all transverse momenta then asymptotic freedom must be preserved as discussed in the last Section. *If $N_f < N_f^{max}$, so that this is not the case, a transverse momentum cut-off must be introduced as part of the analysis.* In the next Section we will see that this cut-off becomes an important dynamical variable.

As is made clear, [4] is at best an outline of a full analysis. We summarize the results first and then elaborate briefly on their origin.

- The presence of light quarks (initially taken to be massless) produces Regge limit infra-red “anomalous” interactions which generate a wide ranging exponentiation of infra-red divergences. In SU(2) gauge theory an overall divergence remains which can be absorbed, as a “condensate”, into the scattering states. The divergence selects out the confinement plus chiral symmetry breaking spectrum of quark hadronic states and is the only high-energy remnant of multi-gluon exchange - there is no Pomeron.

- For SU(3) gauge symmetry broken to SU(2), a Regge pole Pomeron is formed by the SU(2) singlet gluon, which is reggeized, in the background of the condensate and can be identified with the Super-Critical Pomeron of Reggeon Field Theory. If the gauge symmetry is restored to SU(3), the condensate disappears and, straightforwardly when $N_f = N_f^{max}$, the Critical Pomeron is obtained.

(Although it does not appear in [4], we should note that the even signature symmetric octet two gluon reggeon discovered in [7] becomes exchange degenerate with the reggeized gluon as the SU(3) symmetry is restored. It also contributes to the Pomeron and is a crucial component at small momentum transfer. For simplicity we shall make only brief references to this in the following since it is irrelevant to our essential purpose - the discussion of diffractive hard scattering.)

It is, of course, well-known that a cloud of soft photons must be introduced in QED to define gauge-invariant electron states and this manifests itself in high-energy scattering as the divergent Coulomb phase. In a non-abelian theory, with massless quarks, our analysis implies that this divergence problem is dominated by effects related to topological gauge fields and the massless fermion anomaly equation i.e.

$$\partial_\mu j_5^\mu = -N_f F\tilde{F}/16\pi^2 = -N_f \partial_\mu K^\mu /16\pi^2 \quad (5.1)$$

j_5^μ is the color singlet U(1) axial-vector current and K^μ is the winding-number current

$$K^\mu = \frac{g^2}{8\pi^2} \epsilon_{\mu\alpha\beta\gamma} Tr \left[-\frac{2ig}{3} \epsilon_{ijk} A_\alpha^i A_\beta^j A_\gamma^k + A_\alpha^i \partial_\beta A_\gamma^i \right] \quad (5.2)$$

If an infinite momentum hadron has a “cloud of topological glue” around it, the relation between topological winding number and vacuum production of axial quark-antiquark pairs implied by (5.1) leads us to expect corresponding divergences from massless quark intermediate states in our Regge limit analysis. These are the divergences that we find. (In this sense, the anomalous interactions which appear specifically in the Regge limit can be understood as the “perturbative realization” of instanton interactions.) The simplest situation occurs in SU(2) gauge theory and the result is analogous to the solution[10] of the two-dimensional Schwinger model in the presence of the anomaly. That is, there is a vacuum (“winding number”) condensate, that appears in our formalism via the factorization of infra-red divergences, and *no scattering*. For SU(3) gauge theory the high-energy Pomeron can be built up perturbatively on top of an SU(2) vacuum condensate as follows.

With the $SU(3)$ gauge symmetry broken to $SU(2)$, our first approximation to Pomeron exchange in hadron scattering is the Regge pole exchange illustrated in Fig. 5.1. Perturbative exchange of the $SU(2)$ singlet reggeized gluon (the circle indicates the reggeization) between quarks (represented by straight lines) is accompanied by a shadow process - represented by the dashed lines. The shadow process is the effect of massless $SU(2)$ gluon exchanges, interacting via massless quark loops, which produce the infra-red divergence that is absorbed onto the definition of the states. We have used three dashed lines because the simplest divergent gluon configuration involved[4] is three gluons in a color-zero combination, corresponding (loosely) to the three gluon component of the $SU(2)$ winding-number current. (The two gluon component occurs in combination with the even signature octet reggeon referred to above).

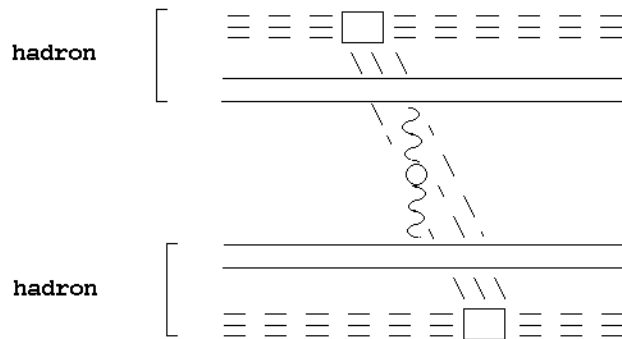


Fig. 5.1 First Approximation to Hadron-Hadron Scattering

We believe that it should also be possible to view the shadow process as the average effect of the exchange of winding number, in the topological sea of each of the scattering hadrons, via instanton interactions. (As we emphasize in [5], when $N_f = N_f^{max}$ there are no infra-red renormalons in the massless theory. This implies that instanton interactions are finite and represent the non-perturbative physics of the theory completely.) We therefore suggest a space-time, or impact parameter picture, of hadron-hadron scattering via Pomeron exchange as illustrated in Fig. 5.2. We picture a hadron at rest as “quarks in a bag”. At large momentum the hadron expands in impact parameter space by virtue of the expansion of the surface of the bag. This is approximated by a classical large impact parameter field (essentially the “wee-parton” component of a hadron) which we can think of as involving a sum over winding number topologies.

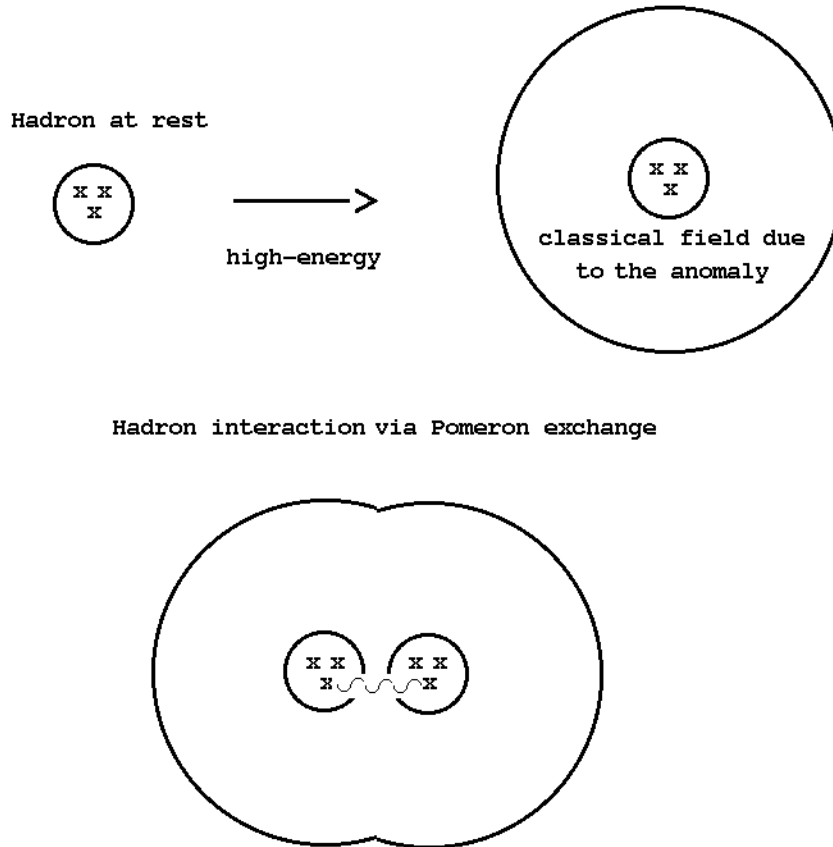


Fig. 5.2 Impact parameter picture of high-energy hadron scattering as perturbative gluon exchange within overlapping topological fields.

It is clear from Fig. 5.2 that hadron scattering at large impact parameter will involve perturbative interactions taking place in the background of the wee-parton fields. However, Fig. 5.1 implies that the condensate also represents a wee-parton contribution to what is exchanged. (Indeed to obtain an even signature Pomeron from gluon exchange, this must be the case.) We think of this as a sum over (instanton) winding-number exchanges coupling the relevant components of the hadron classical fields. If we anticipate that Fig. 5.2 retains some validity after the restoration of $SU(3)$ symmetry, it is clear also that a transfer of $SU(3)$ color in the perturbative interaction can be negated by an (instanton) interaction between the wee-parton fields which carries compensating color. We discuss this point further at the end of the next Section.

The complete Pomeron is built up by multiple massive gluon exchanges and interactions, which also couple to the condensate, as illustrated in Fig. 5.3 *For the restoration of the $SU(3)$ symmetry it is crucial that both the condensate and the*

reggeized gluon are $SU(2)$ singlets but are octets under $SU(3)$ color. The combination of the condensate and the gluon have an $SU(3)$ singlet projection and so form the $SU(3)$ Pomeron. As the full symmetry is restored, the massive gluon becomes massless, the gluons forming the condensate become dynamical, and a Pomeron with unit intercept, the Critical Pomeron, is obtained. In the process the singlet gluon also becomes an important part of a hadron, combining with the condensate gluon configuration to give an $SU(3)$ invariant gluon (or Pomeron) contribution. In this way a hadron regains it's conventional structure, becoming predominantly an $SU(3)$ singlet quark state, combined with a singlet combination of wee-parton gluons. (As we noted above, the $SU(2)$ singlet component of the $SU(3)$ symmetric octet two gluon reggeon[7] also plays a crucial role in the formation of the $SU(3)$ Pomeron).

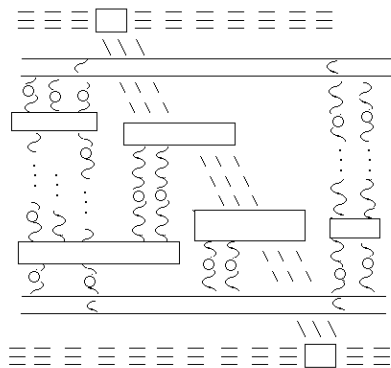


Fig. 5.3 The Full Set of Pomeron Graphs

Finally we note that in a normal perturbative (small impact parameter) interaction there can also be an instanton interaction between the wee-parton fields. As the $SU(3)$ symmetry is restored, a gluon interaction generating the Pomeron must be constrained to carry the same color as the wee-parton interaction, while in a perturbative gluon interaction this need not be the case. As we observed in the Introduction this can be viewed as a simple origin of the factor of 1/8th in the relative cross-sections.

6. POMERON FIELD THEORY

The idea that the strong interaction at high-energy is described by a self-interacting Regge pole - the Pomeron - has a long history. In [5] it is argued that unitarity actually requires that asymptotically rising total cross-sections be described by

the strongly coupled Critical Pomeron. The effective RFT near the Critical Pomeron fixed-point is a simple triple Pomeron theory with the lagrangian

$$\mathcal{L} = \frac{1}{2}\bar{\psi}\overleftrightarrow{\frac{\partial}{\partial y}}\psi - \alpha'_0\nabla\bar{\psi}\nabla\psi + \Delta_0\bar{\psi}\psi - \frac{1}{2}ir_0\left[\bar{\psi}\psi^2 + \bar{\psi}^2\psi\right], \quad (6.1)$$

where $\bar{\psi}$ and ψ are, respectively, field operators creating and destroying Pomerons in impact parameter and rapidity space. r_0 is the triple Pomeron coupling and α'_0 is the slope. For $\Delta_0 \equiv (1 - \alpha_{\mathbb{P}}(0)) \gg 0$, the perturbation expansion defines the theory. The critical behavior occurs at some $\Delta_0 = \Delta_{0C}$ and it can be shown that if there is *no transverse momentum cut-off* then $\Delta_{0C} = -\infty$. With a cut-off Λ_{\perp} , we have

$$\Delta_{0C} = \frac{r_0^2}{\alpha'_0} \ln \frac{r_0^2}{\alpha_0 \Lambda_{\perp}} + 0(r_0^2) < 0. \quad (6.2)$$

To define a (cut-off) theory with $\Delta_0 \ll \Delta_{0C}$, we look for a classical field configuration minimizing the “potential”

$$V(\bar{\psi}, \psi) = \Delta_0\bar{\psi}\psi + \frac{ir_0}{2}(\bar{\psi}^2\psi + \bar{\psi}\psi^2). \quad (6.3)$$

The relevant stationary point is

$$\psi = \bar{\psi} = \frac{2i\Delta_0}{3r_0} \quad (6.4)$$

and the new potential is

$$\tilde{V}(\bar{\psi}, \psi) = \frac{-\Delta_0}{3}\bar{\psi}\psi - \frac{\Delta_0}{3}\bar{\psi}^2 - \frac{\Delta_0}{3}\psi^2 + \frac{ir_0}{2}(\bar{\psi}^2\psi + \bar{\psi}\psi^2) \quad (6.5)$$

giving a perturbation expansion that is stable for $\Delta_0 \ll 0$, i.e. $\alpha_0 \gg 1$. The $\bar{\psi}^2$ and ψ^2 terms are a direct reflection of the presence of a Pomeron condensate.

When the RFT graphs describing this Super-Critical theory are constructed[5] and analysed via reggeon unitarity, we find that there is an odd-signature trajectory degenerate with the Pomeron (both have intercept below one). The odd-signature reggeon couples pairwise to the Pomeron. In detail we find that the reggeon diagrams of the Super-Critical Pomeron theory have just the structure of those described in the last Section, i.e. the reggeon diagrams of QCD with the gauge symmetry broken from SU(3) to SU(2). The Pomeron condensate is identified with the reggeon (wee parton) condensate arising from the infra-red divergences.

If we assume that, in the neighborhood of the critical point, the parameters of QCD map straightforwardly on to those of RFT, a number of conclusions follow. Firstly, although we constructed the Super-Critical phase by using the Higgs mechanism to “break the gauge symmetry”, this is not necessary. The true characterization of the super-critical phase is an exchange degenerate Pomeron together with a Pomeron condensate. In general Λ_\perp is a relevant RFT parameter at the critical point, in addition to $\Delta_0 = (1 - \alpha_0)$. The analysis of the previous Section shows that in the particular case $N_f = N_f^{max}$ the critical point is at $\Lambda_\perp = \infty$. Assuming, as follows from (6.2), that $\Delta_0 < \Delta_{0C}$ corresponds to $\Lambda_\perp < \Lambda_{\perp C}$, it is clear that the Super-Critical phase can also be reached without introducing an additional “Higgs parameter”. The RFT analysis implies that when $N_f = N_f^{max}$ the Super-Critical Pomeron occurs when $\Lambda_\perp < \infty$. When $N_f < N_f^{max}$, the critical point is at a finite value of the transverse momentum cut-off and so the *Super-Critical phase can be introduced by taking $\Lambda_\perp < \Lambda_{\perp C}$.*

Let us consider, therefore, how the Super-Critical Pomeron can occur in an SU(3) invariant manner. It might appear that a simple SU(3) invariant version of our discussion would involve the usual SU(3) winding-number current, in which the ϵ_{ijk} in (5.2) are replaced by the SU(3) structure constants. However, since this is a color zero operator, it is clear that the Pomeron could not appear as gluon exchange in the background of a related condensate. Instead the Super-Critical Pomeron must be associated with interactions of the kind represented by Fig. 5.1 and Fig. 5.2, but with the color of the exchanged gluon and that of the classical hadron fields summed and averaged over. In fact it is well known that instantons are essentially an SU(2) phenomenon[19] and that the simplest topology SU(3) instantons are always confined within an SU(2) subgroup. Our present discussion suggests that dynamical effects of the fermion anomaly must be accounted for before instanton parameters, corresponding to group rotation of the sub-group, are summed over to obtain an SU(3) invariant result. This implies, in particular, that a Super-Critical hadron is an SU(3) triplet in a non-trivial way. That is, the quark component of the hadron is in an octet state corresponding to the SU(3) generator that leaves an initial instanton subgroup unchanged. The “topological glue component” resulting from instanton interactions within that subgroup is then necessary to obtain an overall singlet. The color direction of the octet is summed over as part of the process of summing over instanton parameters.

In the Super-Critical phase, therefore, diffractive scattering can be calculated using a massive gluon. The presence of a transverse momentum cut-off is crucial in

that the mass is generated dynamically by taking the cut-off *below* the critical value. Physically this will automatically be the case if the critical cut-off is above presently accessible transverse momenta. It is interesting to ask what physically sets the scale of this mass. We anticipate that $\Lambda_{\perp C}$ is a function of $N_f^{max} - N_f$, but from (6.2) we see that the cut-off enters only logarithmically in determining how close to criticality the Pomeron is. As a result the gluon mass scale is mainly set by

$$m_g^2 = \frac{\Delta_{0C} - \Delta_0}{\alpha'_0} \sim \frac{r_0^2}{\alpha'_0{}^2} \quad (6.6)$$

We anticipate that $r_0 \sim 1/m_\pi$ and $\alpha'_0 \sim 1/\lambda_{QCD}^2$, in which case

$$m_g \sim \frac{\lambda_{QCD}^2}{m_\pi}$$

In the physical world, as currently explored, we have both $N_f < N_f^{max}$ and of course, simply because of the limitation of finite energy, $\Lambda_\perp < \infty$. It also seems to be established that the “soft Pomeron” is well described phenomenologically[20] by an (unstable) Super-Critical Pomeron expansion with a bare Pomeron intercept $\alpha_{\mathbb{P}}(0) > 1$. Clearly we can consistently assume that we are in the Super-Critical phase. In the RFT framework, therefore, the Super-Critical expansion involving a Pomeron condensate is really the most appropriate way to describe the Pomeron. Although we should note that RFT graphs describe idealised (asymptotic) physical processes in which finite-energy kinematic limitations on sub-energies and transverse momenta are not imposed. Nevertheless, as we have discussed, the Super-Critical graphs are related (albeit in a complicated way) to perturbative QCD processes. Also the gluon mass can be regarded as induced by the very large transverse momentum region (making it sensitive to as yet unexplored higher momentum scales). Consequently the Super-Critical formalism is very likely to be the most appropriate for discussing diffractive hard-scattering processes. This clearly is the point of view we advocate as providing the correct interpretation of the BH picture for deep-inelastic scattering.

We should emphasize that if the theory is truly in the Super-Critical phase then RFT implies that the total cross-section will ultimately fall asymptotically. This can not be consistent with the validity of QCD perturbation theory at large transverse momentum. As we already remarked in the Introduction, we believe that, for full consistency the additional fermion sector, needed to obtain the Critical Pomeron with no transverse momentum cut-off, must eventually enter the theory.

7. DEEP-INELASTIC SCATTERING

Let us return now to the QCD Pomeron in the idealised situation where we have $N_f = N_f^{max}$ and the gauge symmetry is explicitly broken to $SU(2)$. Consider deep-inelastic scattering of a hadron and a photon carrying large (spacelike) Q^2 . Let us compare the forward amplitude giving the total cross-section with the diffractive cross-section. In the calculation of the total cross-section, the mass we have introduced has little impact. Asymptotic freedom exposes the elementary quark-antiquark coupling at large Q^2 and we can suppose also that the kinematics of the final state justify selecting Fig. 1.2 as the dominant perturbative process in the forward amplitude. (As we discussed earlier, this a subtle point since Fig. 1.2 successfully describes far more of the total cross-section than it should.) The gluon involved can be either massless or massive and as the $SU(3)$ gauge symmetry is restored the distinction becomes irrelevant. (Since we are not discussing diffractive scattering, there is no appearance of the condensate). We therefore obtain the usual full QCD contribution of Fig. 1.2 to the total cross-section.

For the diffractive cross-section corresponding to the same quark-antiquark contribution to the final state we obtain Fig. 7.1.

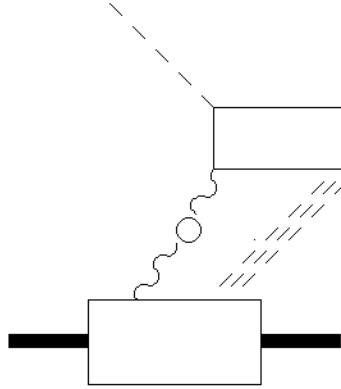


Fig. 7.1 Diffractive Deep-Inelastic Scattering

We could show the photon as having a “topological sea” but this should, of course, be irrelevant at large Q^2 . Therefore we have shown the exchanged infra-red divergent gluons as absorbed by the parton final state without any rescattering on the corresponding component of the photon. Large Q^2 again determines that the pho-

ton couples perturbatively, as we have shown in Fig. 7.1. Now, since the Pomeron is involved, it is necessarily the massive SU(2) singlet reggeized gluon that appears. The box in the lower half of Fig. 7.1 contains the full set of Pomeron graphs having, essentially, the structure illustrated in Fig. 5.3.

The impact parameter picture corresponding to Fig. 7.1 is shown in Fig. 7.2. The elementary photon enters the center of the hadron and produces an additional quark pair. There is a perturbative gluon exchange before the classical field component of the hadron breaks apart in a manner that, as the SU(3) symmetry is restored, compensates for the color exchange.

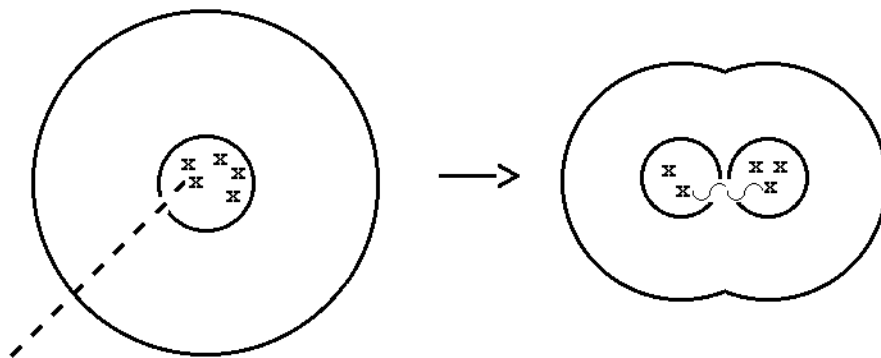


Fig. 7.2 Impact Parameter Picture of Diffractive Deep-Inelastic Scattering

There are two crucial ingredients that lead us to the BH result for the scaling violations - as the SU(3) symmetry is restored. The first is that *the condensate disappears smoothly*. The second is that *asymptotic freedom continues to select the perturbative photon-gluon scattering process* - in the region of the cross-section that is insensitive to m_g^2 . We are, of course, effectively assuming that the deep-inelastic, diffractive, and symmetry restoration limits all commute for this part of the cross-section.

To discuss the x -dependence, let us first make the usual BFKL assumption that the small value of α_s generated by the deep-inelastic limit justifies using *the leading log[1/ ξ] approximation*. Taking the SU(3) symmetry limit and considering the full cross-section for the diffractive excitation of the proton we obtain a simple result. The condensate disappears and, because the reggeization of the gluon is included in the Pomeron, the leading-order BFKL equation appears with the SU(2) singlet gluon still coupled to the quark/antiquark pair. This is illustrated in Fig. 7.3, where K

denotes the BFKL kernel and we have used a simple loop to represent the leading log reggeization. The solution gives the small- x (singular) structure function $g(\xi)$, as required. According to our formalism therefore, it is consistent that the BFKL Pomeron gives the behaviour of parton distributions at small- x , even though it does not describe the physical Pomeron that is responsible for a rapidity gap.

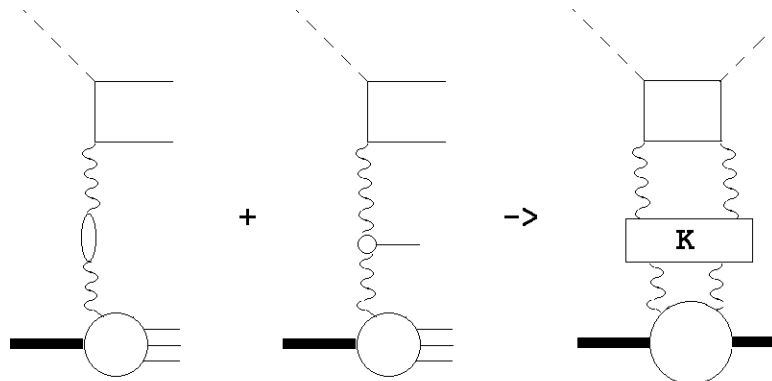


Fig. 7.3 The BFKL equation for the Diffractive Cross-section

In the leading-log approximation, we obtain all the ingredients of the BH cross-section. The normalization is clearly $1/8$ th since the cross-section is specifically given by a single gluon, i.e the gluon that is an $SU(2)$ singlet under the symmetry-breaking procedure, and we could have chosen any of the eight gluons to play this role. To extend the argument beyond leading-logs, we need to discuss the contribution of the symmetric octet two-gluon reggeon[7] to the Super-Critical Pomeron and the eventual emergence of the Critical Pomeron. The self-consistency properties of the Critical Pomeron will again give the scaling property, but with an undetermined relative normalization.

Note that for Fig. 7.1 to make sense it is crucial that the quark-antiquark state can combine with the $SU(2)$ singlet condensate to make a hadronic final state. As the $SU(3)$ symmetry is restored, this requires that it be an $SU(3)$ octet, which of course it is. In this sense it is vital that we consider Fig. 1.2 throughout the full kinematic regime. The normal formulation of the parton model within QCD requires that we separate kinematic configurations in which either the quark or antiquark is relatively soft and sum over related processes to introduce parton distributions. In this case it would not be consistent to introduce the condensate as we have done. In general, for a short-distance parton process to couple to the Pomeron in analogy with our present discussion, we believe it is necessary that the parton state be a quark state

that can similarly be converted to a hadronic final state. (It is possible that the final state could also be a massive $SU(2)$ singlet gluon state, before the gauge symmetry is restored to $SU(3)$. This relates to the issue of whether there are glueballs in the physical spectrum and for the present we will assume that this is not the case.)

For the BH model to be applicable to QCD with just the normal number of quarks, it is very clear from the discussion of this and the previous Section what we must assume to be the case. The effective transverse momentum cut-off imposed by current energies must be below the critical cut-off, so that the Super-Critical Pomeron phase is realised. In addition an approximate form of asymptotic freedom must still be valid to justify using deep-inelastic scattering to expose single gluon exchange. (Of course, this must also be the case to justify the general success of perturbative QCD!!)

If the diffractive deep-inelastic scaling property is to survive asymptotically, we must have full asymptotic freedom combined with the proximity of the Super-Critical phase. This implies we must have the conditions discussed in Section 4, i.e. a higher-mass quark sector must eventually enter the theory.

8. DIFFRACTIVE HARD SCATTERING AT THE TEVATRON

We now consider the case of Super-Critical diffractive W production in hadron-hadron scattering. (We no longer distinguish the explicit breaking of the gauge symmetry from the case of unbroken QCD - in which we simply average over the $SU(2)$ direction.) Assuming the large momenta involved exposes a parton production process, our previous discussion implies there must be a quark-antiquark pair in the final state. Therefore we should consider a process of the form shown in Fig. 8.1 Since the large-momentum quark pair is most likely to be produced by a gluon we expect the leading-order diffractive W production to be via the “gluon-gluon fusion” process illustrated in Fig. 8.2.

We must, however, qualify what is meant by gluon-gluon fusion in our case. In analogy with our discussion of photon-gluon fusion earlier, predominantly one of the quark/anti-quark pair will be relatively soft. The W must be produced by the hard quark(anti-quark) and so the correct parton model description is the gluon-quark (gluon-antiquark) process of Fig. 8.3 with the soft anti-quark (quark) production absorbed in the structure function.

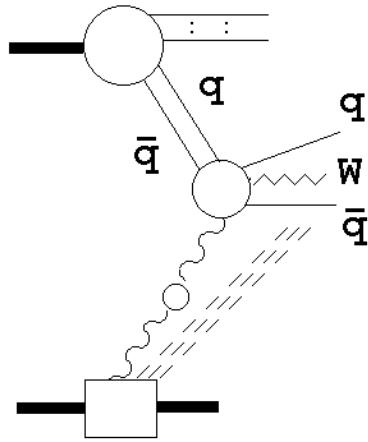


Fig. 8.1 Diffractive W Production

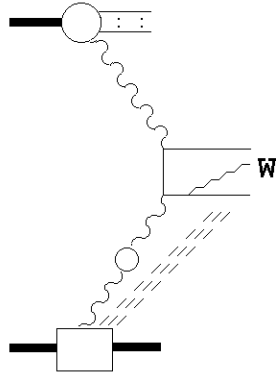


Fig. 8.2 Diffractive W Production via Gluon-Gluon Fusion

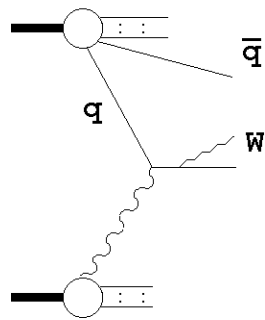


Fig. 8.3 W Production via Quark-Gluon Scattering

As we have emphasized, the role of the additional soft “parton” is crucial in the diffractive process. Therefore the necessity for its presence must be allowed for when

we compute the full parton scattering process that we wish to multiply by $1/8$ th, just as the full deep-inelastic cross-section was used to estimate the cross-section for Fig. 1.2. The effect is that we must, in principle at least, extract from all quark and anti-quark parton distributions (including valence quarks and antiquarks) the contribution due to an intermediate gluon.

Given the increase of the gluon distribution at small- x , the total contribution of gluon-gluon scattering to all processes of the form of Fig. 8.3, *including both valence and sea quarks and antiquarks*, could easily be as high as 10% of the total cross-section. In this case we would expect the diffractive cross-section to be of the order of 1% of the total. This is certainly compatible with, and perhaps even suggested by, the CDF analysis of rapidity-gap W production[12].

As we mentioned above, there is, in our view, a problem with the interpretation of the CDF data in terms of the Ingelman-Schlein Pomeron hard scattering formalism - in which the Pomeron is assumed to have a hadronic structure function. Independently of whether quarks or gluons dominate the Pomeron structure function the Pomeron predominantly scatters off a quark (or antiquark), as illustrated in Fig. 8.4.

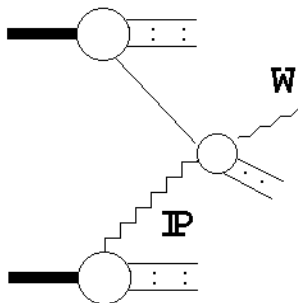


Fig. 8.4 W Production via Pomeron-Quark Scattering

If conventional parton distributions are assumed for the quark scattering off the Pomeron, clearly valence quarks (and antiquarks) will dominate. As a result there should be a strong correlation (in proton-antiproton scattering) between the charge of the W and where it appears on the rapidity axis relative to the rapidity-gap. In fact there is no evidence for such a correlation in the candidate gap events, leading CDF to make the strong conclusion[12] that they see no diffractive W production. Since the multiplicity distribution that would extrapolate to the gap cross-section looks identical for W and jet production, this leads to the even stronger conclusion that they see no diffractive di-jet production.

It is clear that our formalism does not predict the charge/gap correlation that the Ingelman-Schlein approach predicts. In our case, the requirement that the final parton state be consistent with confinement implies that the W is produced from a quark-antiquark pair. This requires in particular that the quark in Fig. 8.4 be accompanied by an antiquark, and in general that either charge is equally likely in any diffractive configuration. The production of the quark pair by a gluon also implies that the W will be produced preferentially close to the rapidity gap. This goes against the expectation that the W will be produced predominantly on the opposite side of the rapidity axis to the gap. Such a correlation is also looked for and apparently not seen in CDF data. Since there are other features of the CDF data for both W and jet production that suggest diffractive production is indeed present, we believe our picture provides a consistent and attractive interpretation of what is seen.

9. COMMENTS AND CONCLUSIONS

We have believed for some time that many fundamental problems are related to understanding the Pomeron in QCD. It involves an intricate combination of short-distance and long-distance dynamics whose solution is surely relevant for any gauge theory. We hope that the striking nature of the scaling behavior displayed in Fig. 1.1 will provoke a serious theoretical effort to determine the implications for the Pomeron. Our work[4] implies these are significant. However, since establishing the picture we have developed would include, in particular, a proof of (high-energy) confinement, it is not too surprising that much remains to be done. Nevertheless developing a simple, consistent, phenomenology should be possible. For this, the connection that we have made with the Buchmüller and Hebecker model[9] ought to be particularly valuable.

Finally we comment on the possibility that deep-inelastic diffractive scaling actually reflects asymptotic properties of the Pomeron and what this implies for new physics at higher energy scales. As we hope our discussion demonstrates, the interplay between deep-inelastic and diffractive scaling actually probes virtual properties of the theory that are related to mass scales yet to be exposed as physical states. We have elaborated elsewhere (e.g. in the summarizing chapter of [4] and in [21]) our belief that the missing states that will ultimately be responsible for the Critical behavior of the Pomeron are a higher color quark sector that is responsible for dynamical symmetry breaking and CP violation in the electroweak sector, as well as a major transformation in strong-interaction physics! There may already be evidence for this physics in Cosmic Rays and in the highest energy hard scattering processes at the Tevatron[21].

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